Calculus Warm Up #11-2

Evaluate the Integrals:

1) $\int \frac{1}{\sqrt{1-x^2}}dx$

2) $\int \frac{x}{\sqrt{1-x^2}}dx$

3) $\int \frac{x}{1+x^4}dx$

4) $\int \frac{x^3}{1+x^4}dx$

HW Questions: p. 516

1. $\int \cos^3 x \sin x \, dx$

3. $\frac{1}{2}\int \sin^5 2x (2\cos 2x \, dx)$ $\longrightarrow$ $u = \sin 2x$

$du = 2\cos 2x$

5. $\int \sin^5 x \cos^2 x \, dx$

7. $\int \cos^2 3x \, dx$

17. $\int \sec^4(5x)\,dx$

$$\int (1+\tan^2 5x)\sec^2 5x\,dx$$

$$\int \sec^2 5x\,dx + \int \tan^2 5x\,\sec^2 5x\,dx$$

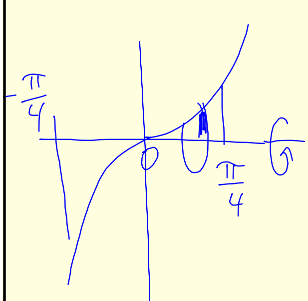
$$u = \tan 5x$$

24. $\int \tan^3 \frac{\pi x}{2} \sec^2 \frac{\pi x}{2}\,dx$

25. $\int \sec^2 x \tan x\,dx$

In Exercises 65 and 66, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the x-axis.

65. $y = \tan x,\ y = 0,\ x = -\frac{\pi}{4},\ x = \frac{\pi}{4}$



$$V = 2\pi\int_0^{\pi/4} (\tan x)^2\,dx$$

$$V = 2\pi\int_0^{\pi/4} (\sec^2 x - 1)\,dx$$

9.4 Trig Substitution

For integrals with radicals that don't fit our basic Integration Formulas:

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin u + C$$

$$\int \frac{1}{a^2+u^2}\,du = \arctan u + C$$

$$\int \frac{1}{|u|\sqrt{u^2-a^2}}\,du = \operatorname{arc\,sec} u + C$$

*Goal: Eliminate the radical using Trig.

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*Goal: Eliminate the radical using Trig.

Example:

$$\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$$

a=3 u=x

Looks like an arcsin:

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin u + C$$

except the x^2 outside

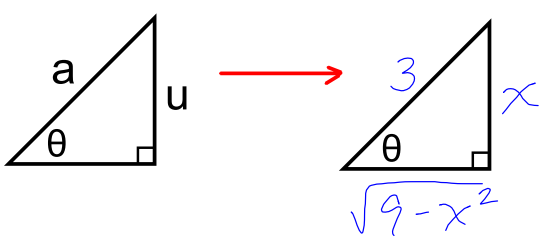
Notice the form of the radical:

$$\sqrt{a^2-u^2}$$

Re-name

Let u = a sinθ

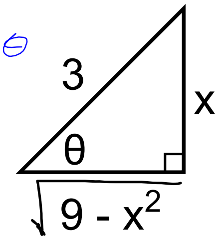
$\sin\theta = \frac{u}{a}$



Bingo!

$$\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$$

Let $u = a\sin\theta$



$$\int \frac{3\cancel{\cos\theta}}{(3\sin\theta)^2(3\cancel{\cos\theta})}\,d\theta$$

$$\frac{1}{9}\int \frac{1}{\sin^2\theta}\,d\theta$$

$$\frac{1}{9}\int \csc^2\theta\,d\theta$$

$$-\frac{1}{9}\cot\theta + C$$

$$\boxed{-\frac{\sqrt{9-x^2}}{9x} + C}$$

$$\cos\theta = \frac{\sqrt{9-x^2}}{3}$$

$$\sqrt{9-x^2} = 3\cos\theta$$

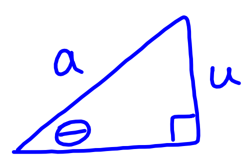
$$x = 3\sin\theta$$

$$dx = 3\cos\theta\,d\theta$$

$\cot \rightarrow \frac{\text{adj}}{\text{opp}}$.

★ $\sqrt{a^2-u^2}$ looks like arcsin

so use $\sin\theta = \frac{u}{a}$



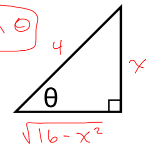
$a > 0$, $u = a\sin\theta$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

You Try: $\boxed{x = 4\sin\theta}$

$$\int \frac{x^2}{\sqrt{16 - x^2}}\,dx$$

$a = 4$ $\quad u = x$



$\cos\theta = \frac{\sqrt{16 - x^2}}{4}$

$\boxed{\begin{array}{c} \sqrt{16 - x^2} = 4\cos\theta \\ dx = 4\cos\theta\, d\theta \end{array}}$

$$\int \frac{(4\sin\theta)^2}{\cancel{4\cos\theta}} \cdot \cancel{4\cos\theta}\, d\theta$$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$$16\int \sin^2\theta\, d\theta$$

$\sin\theta = \frac{x}{4}$

$$\frac{16}{2}\int (1 - \cos 2\theta)\, d\theta$$

$u = 2\theta$

$du = 2\, d\theta$

$$8\left[\theta - \frac{1}{2}\int 2\cos 2\theta\, d\theta\right]$$

$$8\theta - 4(\sin 2\theta) + C$$

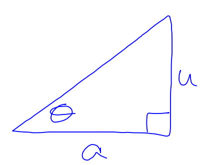
$$8\arcsin\left(\frac{x}{4}\right) - 4(2\sin\theta\cos\theta) + C$$

$$8\arcsin\left(\frac{x}{4}\right) - \cancel{8}^{\,2}\left(\frac{x}{\cancel{4}}\right)\frac{\sqrt{16 - x^2}}{\cancel{4}} + C$$

$$8\arcsin\left(\frac{x}{4}\right) - \frac{x\sqrt{16 - x^2}}{2} + C$$

for $\sqrt{a^2 + u^2}$ $\quad$ let $u = a\tan\theta$

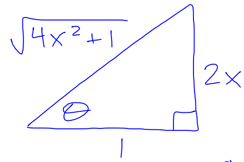
$\tan\theta = \frac{u}{a}$



Example:

$$\int \frac{1}{\sqrt{4x^2 + 1}}\,dx$$

$u = 2x$ $\quad a = 1$



$\cos\theta = \frac{1}{\sqrt{4x^2 + 1}}$

$$\int \cos\theta \cdot \frac{1}{2}\sec^2\theta\, d\theta$$

$\tan\theta = 2x$

$x = \frac{1}{2}\tan\theta$

$dx = \frac{1}{2}\sec^2\theta\, d\theta$

$$\frac{1}{2}\int \frac{1}{\sec\theta} \cdot \sec^2\theta\, d\theta$$

$$\frac{1}{2}\int \sec\theta\, d\theta$$

$$\frac{1}{2}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$\frac{1}{2}\ln\left|\sqrt{4x^2 + 1} + 2x\right| + C$$

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1 - 4, 9 - 12, 15